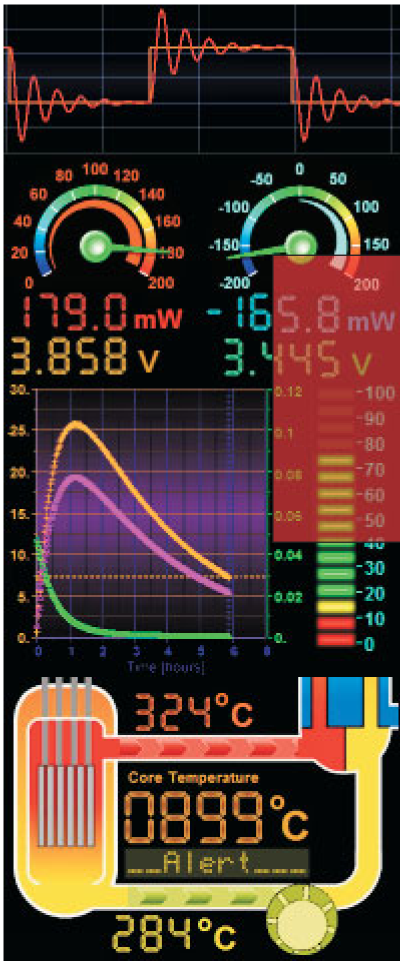
LabRecon - Getting Started with Simulation

LabRecon serves as a rich graphical simulation environment. By wiring mathematical functions together models can be created to simulate dynamic systems. The resulting curves can be presented on the Panel.



LabRecon

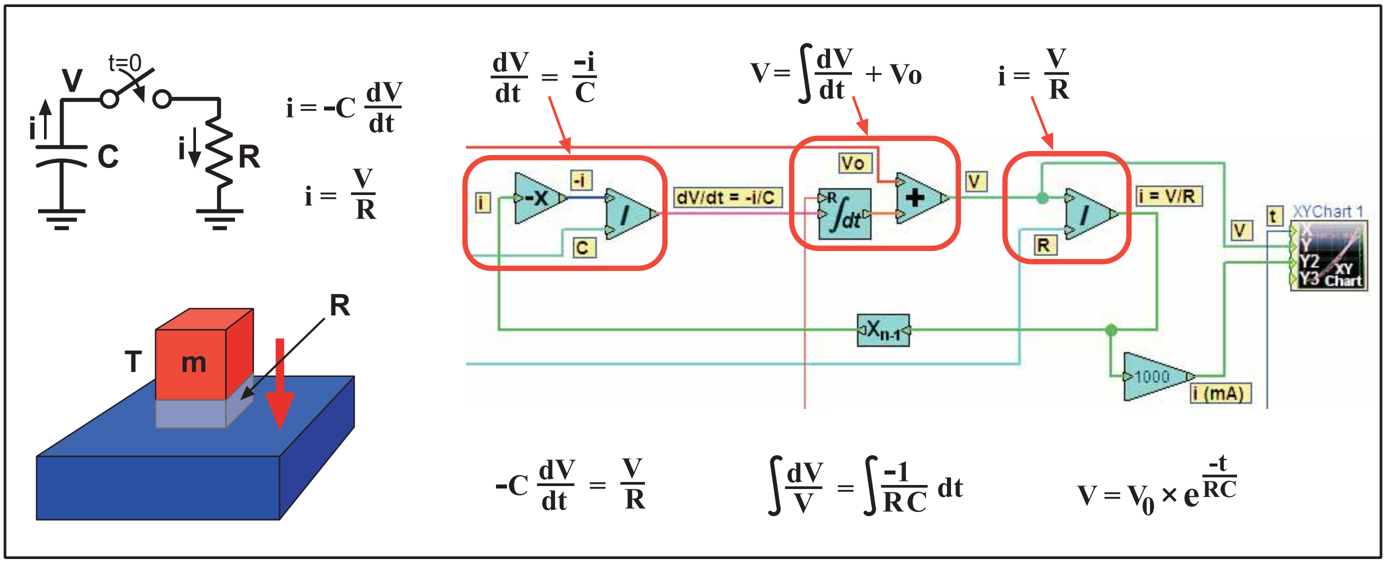
Software and Hardware for Measurement, Control and Simulation

This document will introduce various systems, showing how electrical circuits can serve as a model for thermal or mechanical dynamics.

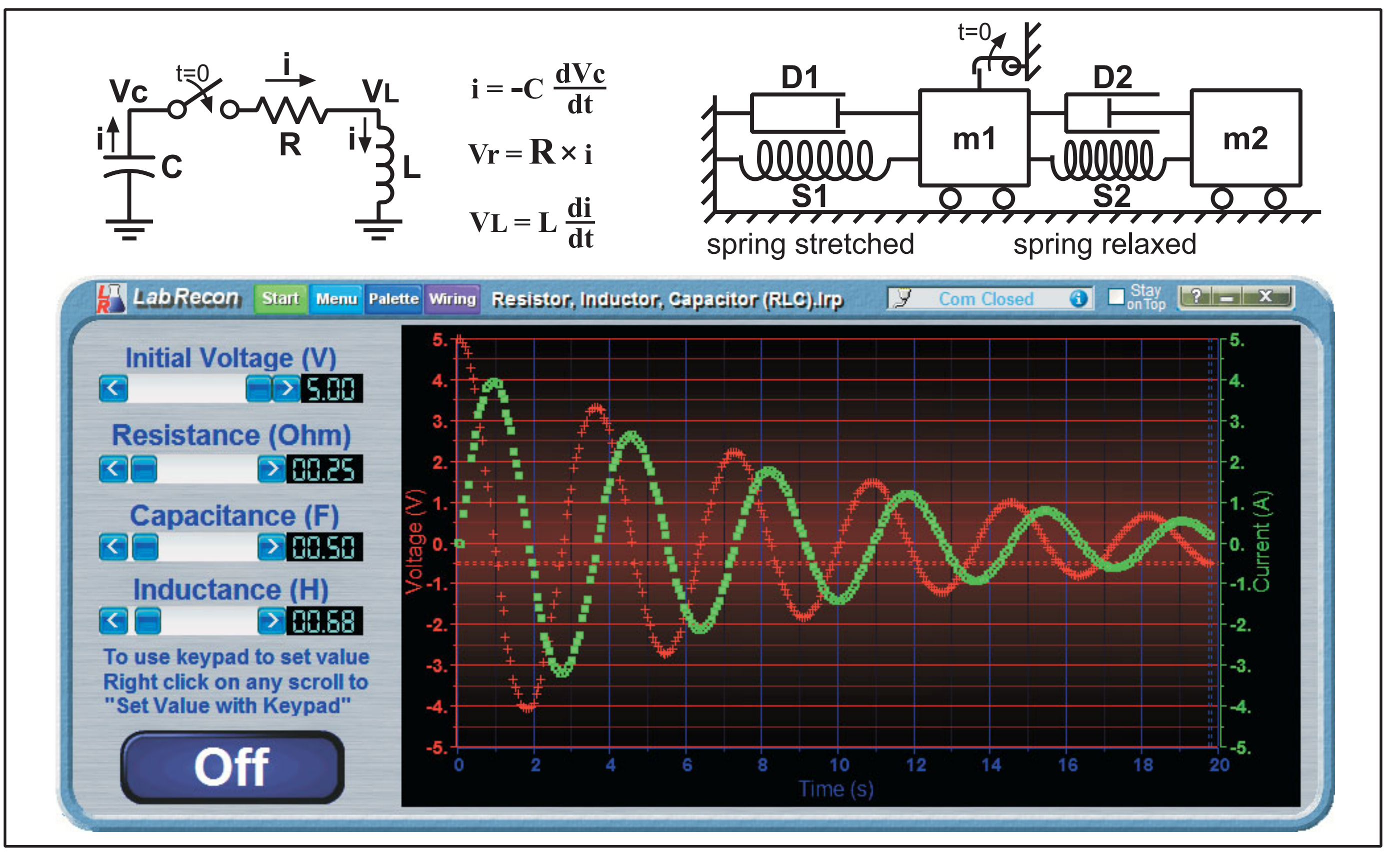
The use of LabRecon to solve the governing differential equation and present the system response is then demonstrated.

The example projects can be downloaded from www.LabRecon.com.

The solution of the differential equation is then derived mathematically to show how it equates to the response determined by LabRecon.



collection of graphics for 1st order systems

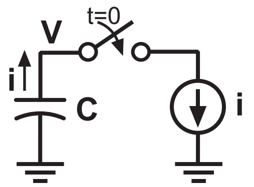


collection of graphics for 2nd order systems

Constant Current Capacitor Discharge (1st order system)

The below circuit depicts a capacitor discharged at a constant current.

The capacitor has an initial voltage, Vo, which starts to decrease when the switch closes.

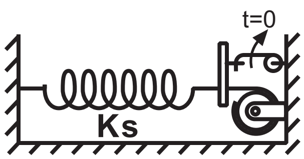


$$i = -C\frac{dV}{dt}$$

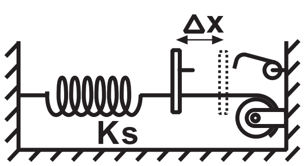
current = -1 × capacitance × rate of voltage change

negative sign needed since current is shown as flowing out of the capacitor

A possible mechanical analogy would include a spring (initially stretched) connected to pulley on a motor. When the hook is released the motor turns the pulley at a constant speed to simulate a constant current. The capacitor has an initial voltage, Vo, which starts to decrease when the switch closes.



spring stretched

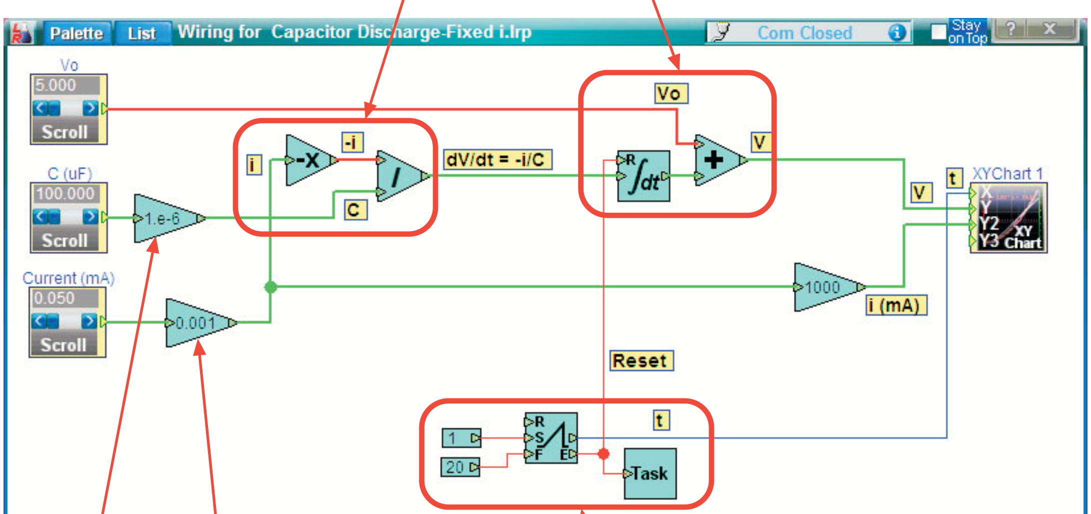


spring relaxed (rest position)

Mechanical		Electrical
force	↔	voltage
position	↔	charge
velocity	↔	current
spring	↔	capacitor
pulley on motor	↔	current source
spring constant	↔	1/capacitance
initial spring force	↔	initial capacitor voltage

Below is the model created on the LabRecon wiring diagram.

$$\frac{-i}{C} = \frac{dV}{dt} \qquad V = \int \frac{dV}{dt} + Vo$$



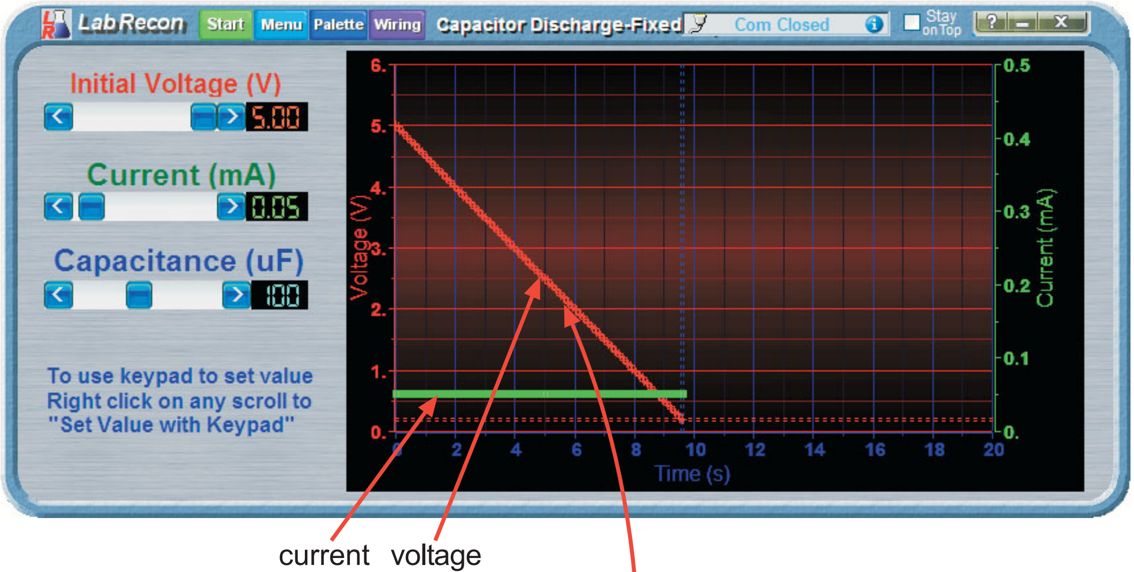
Converts resistance value Kohm to Ohm

Converts capacitance value uF (microFarad) to F (Farad)

Ramp function is used to create time value, t, to plot values on XY Chart. The constant shown set as 1 determines the ramp rate. The lower constant shown as 20 sets the time period. The Task function clears the chart at the end of the time period. The Reset signal also resets the Integration function.

Constant Current Capacitor Discharge (continued)

Below is the Panel showing the resultant capacitor voltage and current curves.



The model created in LabRecon solved the differential equation numerically. Alternatively, the differential equation can be solved to arrive at a solution equation using **Separation of Variables.**

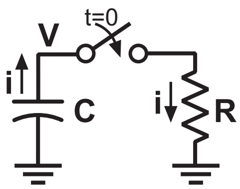
$i = -C\dfrac{dV}{dt}$	from previous page
$dV = \dfrac{-1}{C}\, i\, dt$	dividing each side by **-C** multiplying each side by **dt**
$\int dV = \int \dfrac{-1}{C}\, i\, dt$	integrating each side
$V = \dfrac{-1}{C}\, i\, t + c$	result including integration constant integration constant from each side combined
$V_0 = \dfrac{-1}{C}\, 0\, t + c$	evaluating for **t=0**
$V_0 = c$	
$V = \dfrac{-1}{C}\, i\, t + V_0$	substituting **Vo** (initial voltage) for **c**

The resulting voltage curve has a fixed slope of $\dfrac{-1}{C}\, i$

Decreasing the capacitance or increasing the current will result in a steeper slope and a quicker discharge.

Variable Current Capacitor Discharge (1st order system)

The below circuit depicts a capacitor discharged through a resistor. The capacitor has an initial voltage, Vo, which starts to decrease when the switch closes. The current is not fixed and changes as the voltage across the resistor changes. This system model is characterized as 1st order because its differential equation includes a 1st derivative.



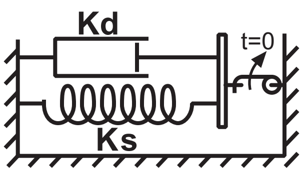
$$i = -C\frac{dV}{dt}$$

current = -1 × capacitance × rate of voltage change
negative sign needed since current is shown as flowing out of the capacitor

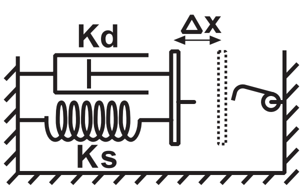
$$i = \frac{V}{R}$$

current = $\frac{\text{voltage}}{\text{resistance}}$

A possible mechanical analogy would include a spring (initially stretched) connected to a dashpot. When the hook is released the force from the spring is equal to that of the dashpot. The dashpot force results from fluid friction whereas its force is not constant, but dependent on the velocity of its piston.



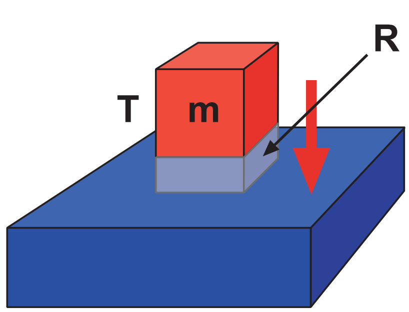
spring stretched



spring relaxed
(rest position)

Mechanical		Electrical
force	↔	voltage
position	↔	charge
velocity	↔	current
spring	↔	capacitor
dashpot	↔	resistor
spring constant	↔	1/capacitance
dashpot constant	↔	resistance
initial spring force	↔	initial capacitor voltage

A possible thermal analogy would include an object (at an initial higher temperature relative to that of a much larger object). When the two objects come into thermal contact heat will flow at a rate determined the by temperature difference and the thermal resistance of the interface between the objects.



Thermal		Electrical
temperature difference	↔	voltage
object mass	↔	capacitance
thermal resistance	↔	resistance
heat flow	↔	current

The Wiring diagram and resultant chart is shown on the following pages.

Variable Current Capacitor Discharge (continued)

The below Wiring Diagram was created to model the capacitor-resistor system.

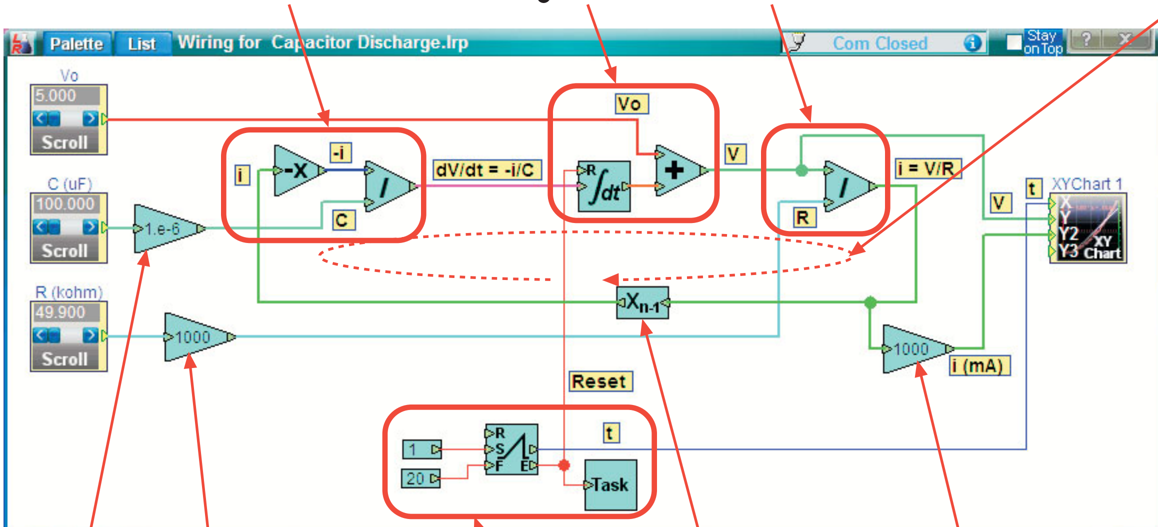
A loop has been employed to handle the variable current, which decreases as the voltage across the resistor decreases. No loop was employed in the previous capacitor-current source example because the capacitor current was constant as the voltage across the resistor changes.

This system model is characterized as 1st order because its differential equation includes a 1st derivative.

Below is the model created on the LabRecon wiring diagram.

$$\frac{dV}{dt} = \frac{-i}{C} \qquad V = \int \frac{dV}{dt} + Vo \qquad i = \frac{V}{R}$$

Loop is needed because current is not constant.



Converts resistance value Kohm to Ohm

Converts current value A (Amperes) to mA (milliAmperes)

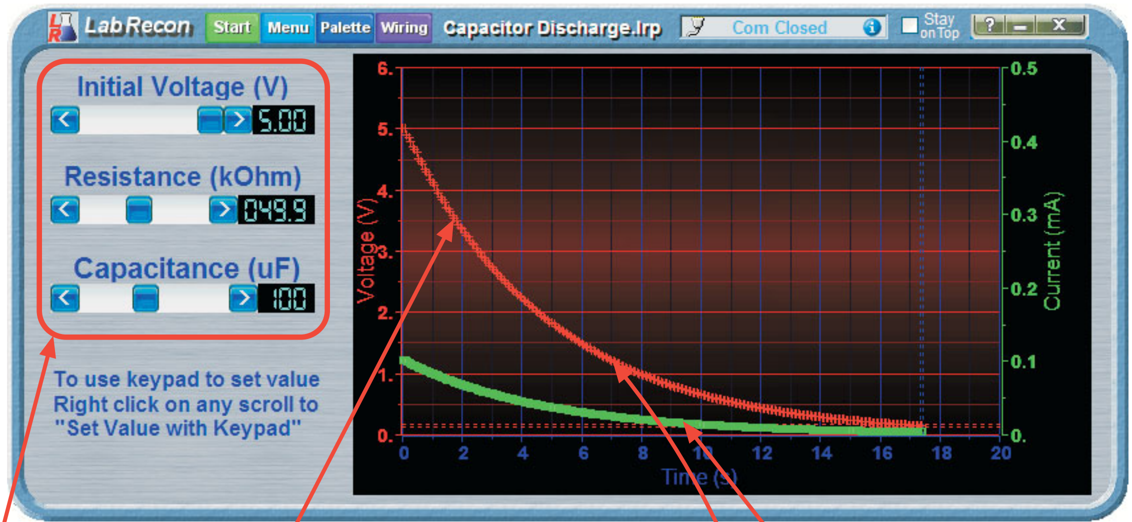
Converts capacitance value uF (microFarad) to F (Farad)

The Shift function allows a value to be looped back.

Ramp function is used to create time value, t, to plot values on XY Chart. The constant shown set as 1 determines the ramp rate. The lower constant shown as 20 sets the time period. The Task function clears the chart at the end of the time period. The Reset signal also resets the Integration function.

Variable Current Capacitor Discharge (continued)

Below is the Panel showing the resultant capacitor voltage and current curves.



Controls to set model parameters

Voltage
starts at the **initial voltage**
decreases **exponentially** to zero

Current
starts at the **initial voltage / resistance**
decreases **exponentially** to zero

The model created in LabRecon solved the differential equation numerically. Alternatively, the differential equation can be solved to arrive at a solution equation using **Separation of Variables.**

$i = -C\frac{dV}{dt} \qquad i = \frac{V}{R}$ from previous page

$-C\frac{dV}{dt} = \frac{V}{R}$ substituting for **i**

$\frac{dV}{dt} = \frac{-1}{RC}V$ dividing each side by **-C**

$\frac{dV}{V} = \frac{-1}{RC}dt$ dividing each side by **V**
multiplying each side by **dt**

$\int\frac{dV}{V} = \int\frac{-1}{RC}dt$ integrating each side

$\ln(V) = \frac{-t}{RC} + c$ result including integration constant
integration constant from each side combined

$e^{\ln(V)} = e^{\left(\frac{-t}{RC}+c\right)}$ natural exponentiating each side

$V = e^{\frac{-t}{RC}} \times e^{c}$ expanding right side

$V = V_0 \times e^{\frac{-t}{RC}}$ substituting **Vo** (initial voltage)

evaluating for **t=0**

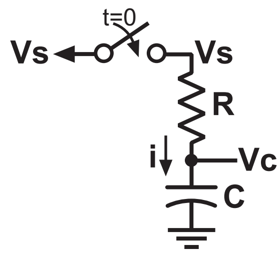
$$V = e^{\frac{-t}{RC}} \times e^{c}$$

$$V_0 = e^{\frac{-0}{RC}} \times e^{c}$$

$$V_0 = 1 \times e^{c}$$

Capacitor Charging (1st order system)

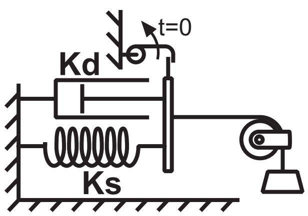
The below circuit depicts a capacitor charged to a voltage through a resistor. The evaluation assumes a capacitor with an initial voltage of zero, which starts to charge when the switch closes. Eventually the capacitor voltage, **Vc**, reaches the supply voltage, **Vs**.



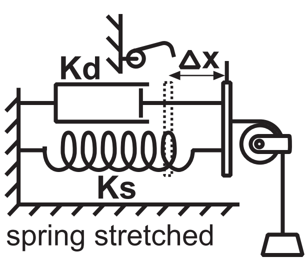
$$i = \frac{Vs - Vc}{R} \qquad \text{current} = \frac{\text{supply voltage} - \text{capacitor voltage}}{\text{resistance}}$$

$$i = C\frac{dVc}{dt} \qquad \text{current} = \text{capacitance} \times \text{rate of voltage change}$$

A possible mechanical analogy would include a spring (initially relaxed) connected to a dashpot. When the hook is released a constant force from the pulley stretches the spring until the force on the spring equals that from the pulley.



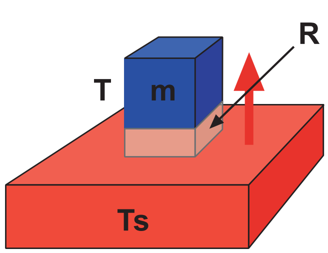
spring relaxed
(rest position)



spring stretched

Mechanical		Electrical
force	⟷	voltage
position	⟷	charge
velocity	⟷	current
spring	⟷	capacitor
dashpot	⟷	resistor
spring constant	⟷	1/capacitance
dashpot constant	⟷	resistance
force from pully	⟷	supply voltage

A possible thermal analogy would include an object (at an initial lower temperature relative to that of a much larger object). When the two objects come into thermal contact heat will flow at a rate determined the by temperature difference and the thermal resistance of the interface between the objects.



Thermal		Electrical
temperature difference	⟷	voltage
object mass	⟷	capacitance
thermal resistance	⟷	resistance
heat flow	⟷	current
source temperature	⟷	supply voltage

Capacitor Charging (continued)

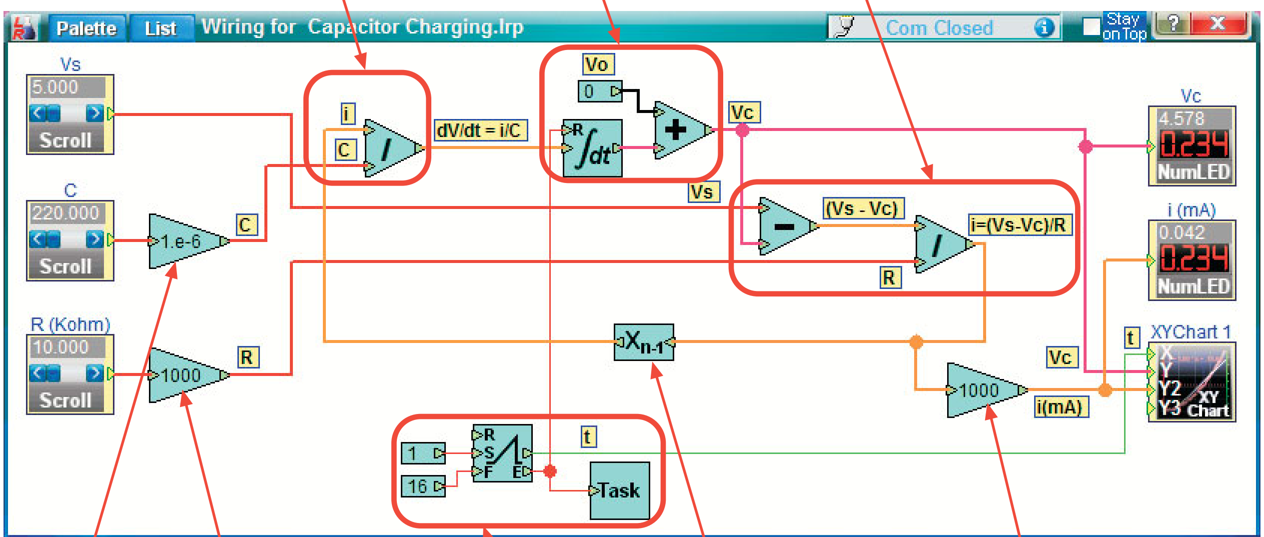
The below Wiring Diagram was created to model the capacitor-resistor system.

A loop has been employed to handle the variable current, which decreases as the voltage across the resistor decreases. No loop was employed in the previous capacitor-current source example because the capacitor current was constant.

as the voltage across the resistor changes.

This system model is characterized as 1st order because its differential equation includes a 1st derivative.

$$\frac{dVc}{dt} = \frac{i}{C} \qquad V = \int \frac{dVc}{dt} + Vo \qquad i = \frac{Vs - Vc}{R}$$



Converts resistance value Kohm to Ohm

Converts capacitance value uF (microFarad) to F (Farad)

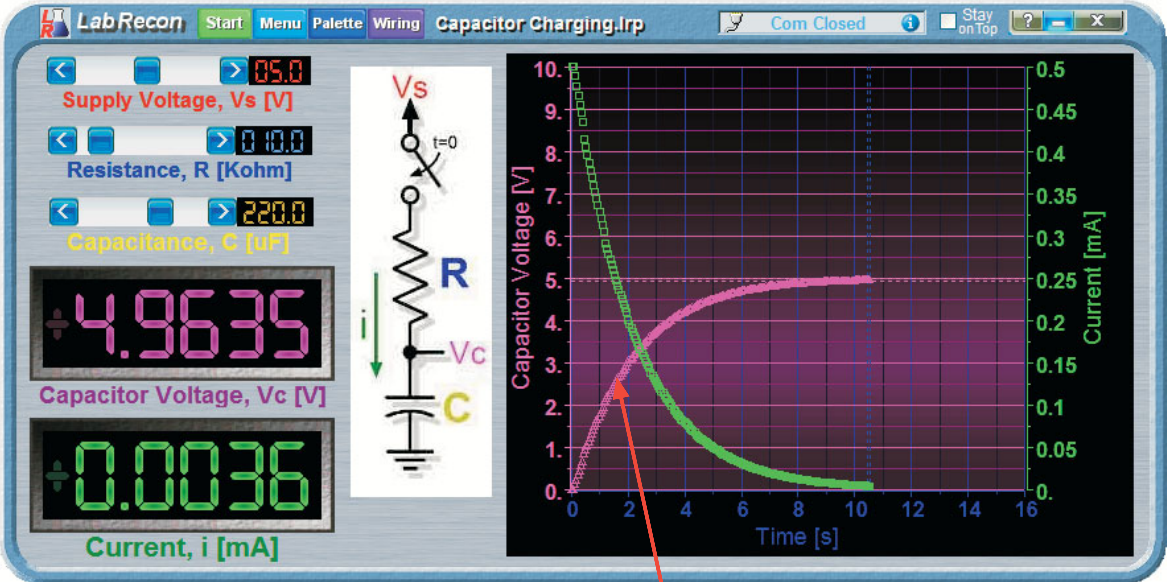
Converts current value A (Amperes) to mA (milliAmperes)

The Shift function allows a value to be looped back.

Ramp function is used to create time value, t, to plot values on XY Chart. The constant shown set as 1 determines the ramp rate. The lower constant shown as 20 sets the time period. The Task function clears the chart at the end of the time period. The Reset signal also resets the Integration function.

Capacitor Charging (continued)

Below is the Panel showing the resultant capacitor voltage and current curves.



The model created in LabRecon solved the differential equation numerically. Alternatively, the differential equation can be solved to arive at a solution equation using **Separation of Variables.**

$i = C\frac{dVc}{dt}$ $\quad i = \frac{Vs - Vc}{R}$ from previous page

$C\frac{dVc}{dt} = \frac{Vs - Vc}{R}$ substituting for **i**

$\frac{dVc}{dt} = \frac{Vs - Vc}{RC}$ dividing each side by **C**

$\frac{dVc}{Vs - Vc} = \frac{1}{RC}dt$ dividing each side by (**Vs - Vc**) multiplying each side by **dt**

$\int\frac{dVc}{Vs - Vc} = \int\frac{1}{RC}dt$ integrating each side

$\int\frac{-du}{u} = \int\frac{1}{RC}dt$ u substitution $u = Vs - Vc$, $du = -dVc$

$-\ln(u) = \frac{t}{RC} + c$ result including integration constant
integration constant from each side combined

$-\ln(Vs - Vc) = \frac{t}{RC} + c$ substituting **u** back

$\ln(Vs - Vc) = \frac{-t}{RC} - c$ multiplying each side by **-1**

$e^{\ln(Vs-Vc)} = e^{\left(\frac{-t}{RC} - c\right)}$ natural exponentiating each side

$Vs - Vc = \frac{e^{\frac{-t}{RC}}}{e^{c}}$ expanding right side

$Vc = Vs - \frac{1}{e^{c}} \times e^{\frac{-t}{RC}}$ solving for **Vc**

evaluating for **t=0**

$Vc = Vs - \frac{1}{e^{c}} \times e^{\frac{-0}{RC}}$

$Vc_0 = Vs - \frac{1}{e^{c}} \times 1$

$\frac{1}{e^{c}} = Vs - Vc_0$

substituting for constant $\frac{1}{e^{c}}$

$Vc = Vs - \frac{1}{e^{c}} \times e^{\frac{-t}{RC}}$

$Vc = Vs - (Vs - Vc_0)\, e^{\frac{-t}{RC}}$

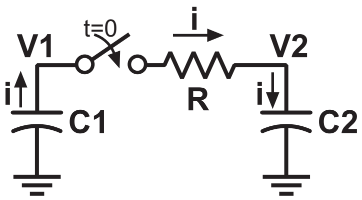
$Vc = Vs\,(1 - (1 - Vc_0)e^{\frac{-t}{RC}})$

$Vc = Vs\,(1 - (1)e^{\frac{-t}{RC}})$

$Vc = Vs\,(1 - e^{\frac{-t}{RC}})$

Capacitor Charge Sharing (1st order system)

The below circuit depicts a capacitor discharged into a 2nd capacitor through a resistor. Each capacitor has an initial voltage which starts to change when the switch closes. Eventually the circuit reaches equilibrium and the two capacitor voltages will be equal.



$$i = -C1\frac{dV1}{dt}$$ current = -1 × capacitance × rate of voltage change

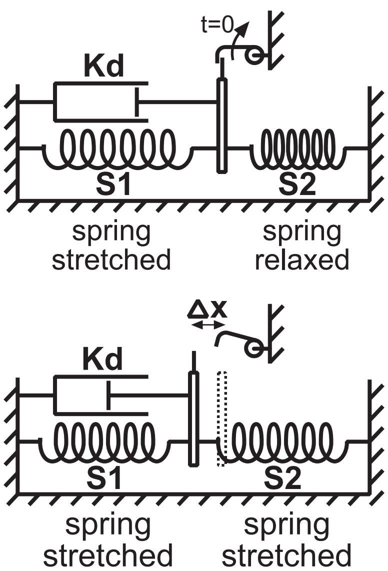
negative sign needed since current is shown as flowing out of the capacitor

$$i = C2\frac{dV2}{dt}$$ current = capacitance × rate of voltage change

$$i = \frac{V1-V2}{R}$$ current = $\frac{\text{voltage difference across resistor}}{\text{resistance}}$

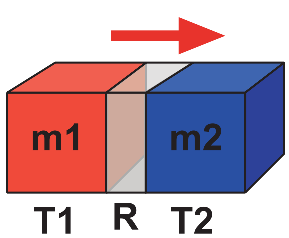
A possible mechanical analogy would be comprised of two springs (one initially stretched and one initially relaxed) connected to each other and a dashpot. When the hook is released the forces reach equilibrium spring 1 becoming less stretched and spring 2 becoming stretched.

In this example the final equilibrium is shown with both springs stretched to the same length. However, if the spring constant is different for each spring, the springs may not have the same final length.



Mechanical		Electrical
force	⟷	voltage
position	⟷	charge
velocity	⟷	current
spring	⟷	capacitor
dashpot	⟷	resistor
spring constant	⟷	1/capacitance
initial spring force	⟷	initial capacitor voltage

A possible thermal analogy would comprise two objects (one at an initial higher temperature relative to the other). When the two objects come into thermal contact heat will flow at a rate determined the by temperature difference and the thermal resistance of the interface between the objects.

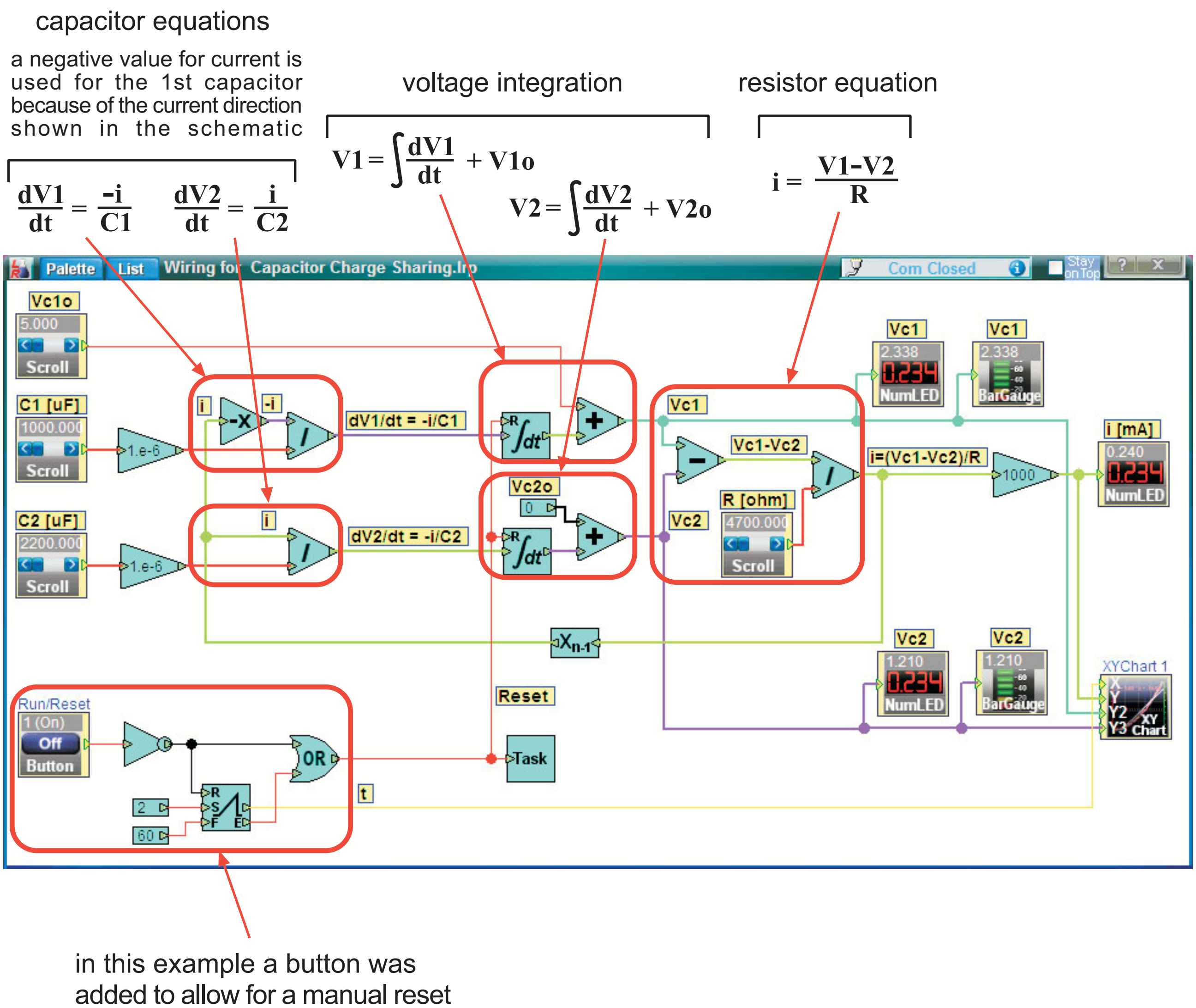


Thermal		Electrical
temperature difference	⟷	voltage
object mass	⟷	capacitance
thermal resistance	⟷	resistance
heat flow	⟷	current

continued on next page

Capacitor Charge Sharing (continued)

Below is the model created on the LabRecon wiring diagram.



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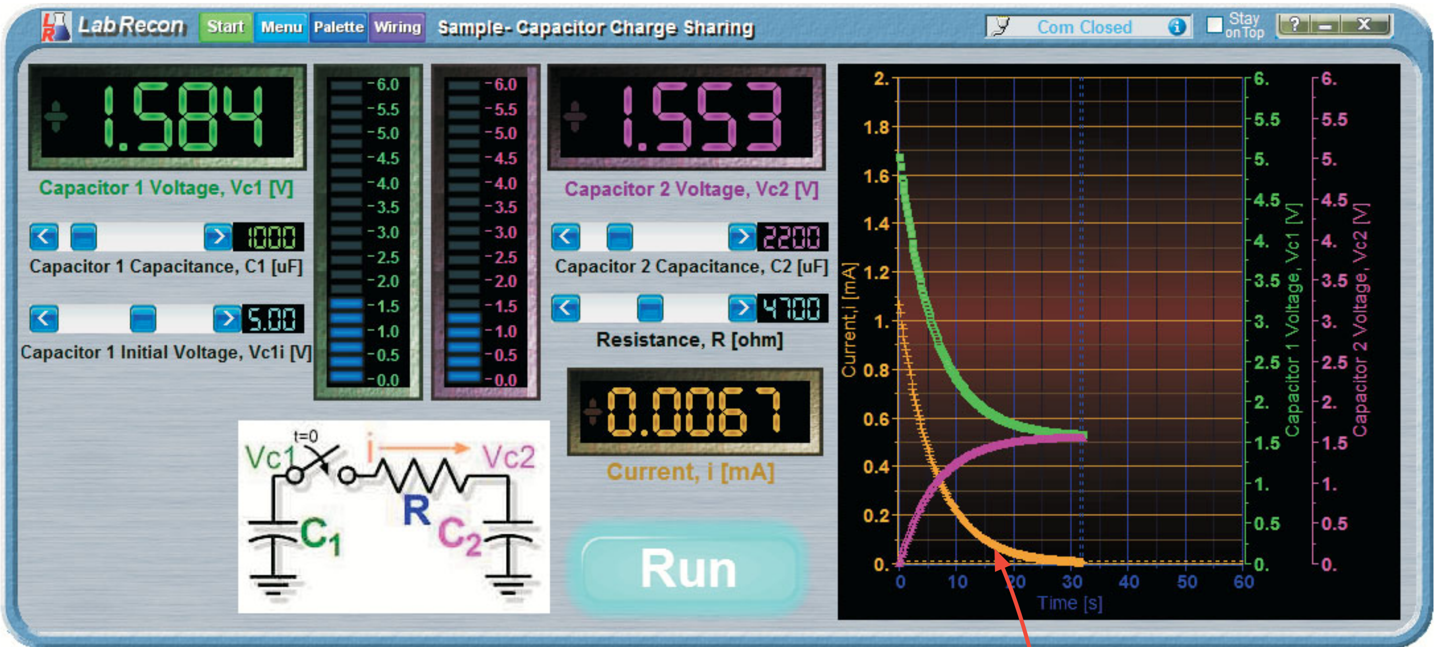
Capacitor Charge Sharing (continued)

Below is the Panel showing the resultant capacitor voltage and current curves.



The model created in solved the differential equation numerically. Alternatively, the differential equation can be solved to arrive at a ution equation using **Separation of Variables.**

$i = -C1\frac{dV1}{dt}$ $\quad i = C2\frac{dV2}{dt}$ $\quad i = \frac{V1-V2}{R}$ from previous page

$\frac{dV1}{dt} = \frac{-i}{C1}$ $\quad \frac{dV2}{dt} = \frac{i}{C2}$ rearranging capacitor equations

$\frac{di}{dt} = \frac{1}{R}\left(\frac{dV1}{dt} - \frac{dV2}{dt}\right)$ time derivative of resistor equation

$\frac{di}{dt} = \frac{1}{R}\left(\frac{-i}{C1} - \frac{i}{C2}\right)$ substitutioning capacitor equations

$\frac{di}{dt} = \frac{-i}{R}\left(\frac{1}{C1} + \frac{1}{C2}\right)$ grouping **i**s

$\frac{di}{i} = \frac{-1}{R}\left(\frac{1}{C1} + \frac{1}{C2}\right)dt$ dividing each side by **i**
multiplying each side by **dt**

$\int\frac{di}{i} = \int\frac{-1}{R}\left(\frac{1}{C1} + \frac{1}{C2}\right)dt$ integrating each side

$\ln(i) = \frac{-1}{R}\left(\frac{1}{C1} + \frac{1}{C2}\right)t + C$ result including integration constant
integration constant of each side is combined

$e^{\ln(i)} = e^{\left(\frac{-1}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)t + C\right)}$ natural exponentiation of each side

$i = e^c \times e^{\left(\frac{-1}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)t\right)}$ expanding and rearranging

$i = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$ substituting for e^c

evaluating for **t=0**

$i = e^c \times e^{\left(\frac{-1}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)0\right)}$

$\frac{V1_0 - V2_0}{R} = e^c \times e^0$

$\frac{V1_0 - V2_0}{R} = e^c$

continued on next page

Capacitor Charge Sharing (continued)

Below a substitution is made to show the capacitance values represented as the resultant capacitance of the two capacitors in series.

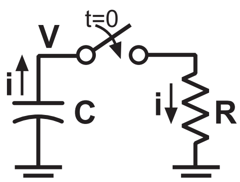
$$i = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$ equation from previous page

$$i = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R\left(\frac{1}{\frac{1}{C1}+\frac{1}{C2}}\right)}\right)}$$ exponent rearranged to reorganize capacitor values

$$C1\&C2 = \frac{1}{\frac{1}{C1}+\frac{1}{C2}}$$ equation for resultant capacitance of the two capacitors in series

$$i = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R(C1\&C2)}\right)}$$ the RC time constant can be determined using $R\times(C1\ \&\ C2)$

An alternative method to find the solution equation for current would be to evaluate a circuit comprising one capacitor as the simpler RC circuit. The equivalent capacitance would be that of both capacitors in series and the initial voltage would be the difference between the two initial capacitor voltages.



$$V_0 = V1_0 - V2_0$$

$$C = C1\ \&\ C2$$

$$i = \frac{V_0}{R} \times e^{\frac{-t}{RC}}$$

To determine the equation for the voltage on a capacitor the equation for the particular capacitor can be used to substitute for i. This is shown below for capacitor C1.

$$i = -C1\frac{dV1}{dt} \qquad i = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$

$$-C1\frac{dV1}{dt} = \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$

$$\frac{dV1}{dt} = \frac{-1}{C1} \times \frac{V1_0 - V2_0}{R} \times e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$

$$V1 = \frac{-1}{C1} \times \frac{V1_0 - V2_0}{R} \times \frac{-1}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right) e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$

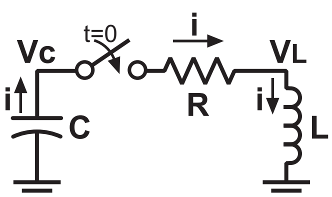
$$V1 = \frac{V1_0 - V2_0}{R} \times \frac{-1}{C1\times R}\left(\frac{1}{C1}+\frac{1}{C2}\right) e^{\left(\frac{-t}{R}\left(\frac{1}{C1}+\frac{1}{C2}\right)\right)}$$ voltage curve for capacitor 1

LRC Inductor, Capacitor, Resistor Circuit (2nd order system)

The below circuit depicts a LRC circuit whereas a capacitor discharges into an inductor through a resistor when the switch is closed. The inductor stores the energy from the capacitor in its magnetic field. The circuit may then oscillate as electrical charge passes back and forth between the capacitor and the inductor. The oscillation will decay as power is dissipated in the resistor.

$$i = \frac{dQ}{dt}$$

Since current is defined as the rate of charge flow, the 1st column of equations are rewritten in terms of charge, **Q**.



$$i = -C\frac{dVc}{dt} \qquad Vc = \frac{-Q}{C}$$

voltage = -1 $\frac{\text{charge}}{\text{capacitance}}$ — negative sign needed since current is shown as flowing out of the capacitor

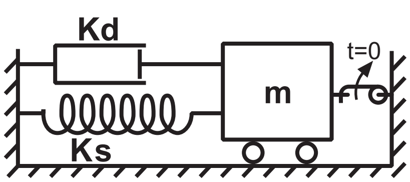
$$Vr = R \times i \qquad Vr = R\frac{dQ}{dt}$$

voltage = resistance × current

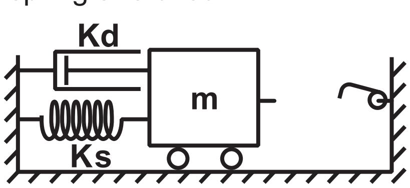
$$VL = L\frac{di}{dt} \qquad VL = L\frac{d^2Q}{dt^2}$$

voltage = inductance × rate of current change

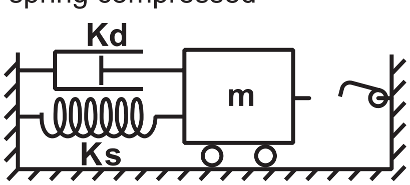
A possible mechanical analogy would be comprised of a spring (initially streched) connected to a dashpot and a mass. When the hook is released the force of the spring accelerates the mass. The momentum of the mass may then overshoot the equilibrium position, thus compressing the spring. The compressed spring can then reverse the direction of the mass. A resultant oscillation may continue, whereas energy is being passed back and forth between the spring and the mass.
The conditions under which the system may oscillate is dependent on the parameters. The dashpot is responsible for damping or preventing the oscillation. If there were no dashpot the system would theoretically oscillate forever.



spring stretched



spring compressed



spring relaxed (rest position)

Mechanical		Electrical
force	↔	voltage
position	↔	charge
velocity	↔	current
spring	↔	capacitor
dashpot	↔	resistor
mass	↔	inductor
spring constant	↔	1/capacitance
initial spring force	↔	initial capacitor voltage

Before **t=0** the capacitor has an initial voltage and the current flow, **i**, is zero. Similarly for the mechanical system, before **t=0** there is an initial force on the mass (from the spring) and there is no velocity.

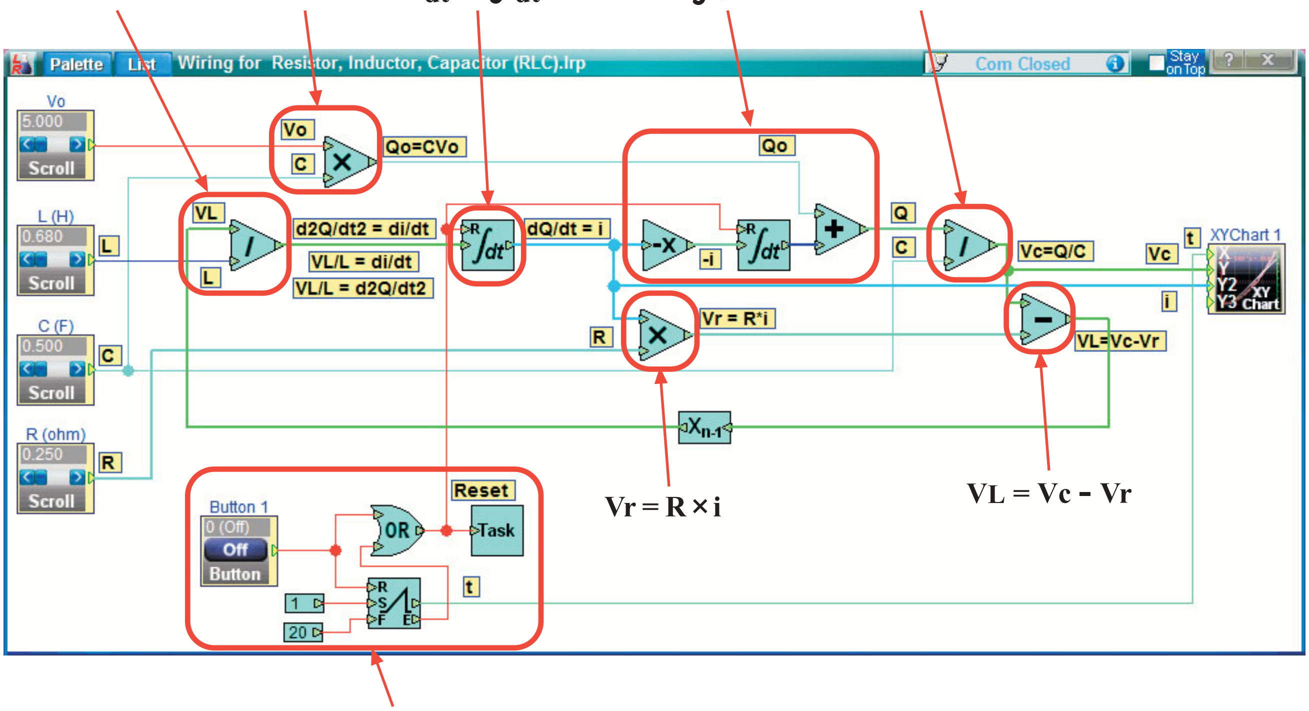
The dashpot is equivalent to the resistor and either element dissipates power to dampen the system. If either the dashpot was removed or the resistor shorted, the system would never stop oscillating, assuming zero friction or ideal components.

It is impractical to build a LRC circuit that will oscillate at a low frequency such as demonstrated in the LabRecon simulation. This limitation is mostly due to the internal resistance of inductors. A typical inductor with a value close to will have a resistance of several hundred ohms. This unwanted resistance will dissipate too much power to allow the circuit to oscillate.

continued on next page

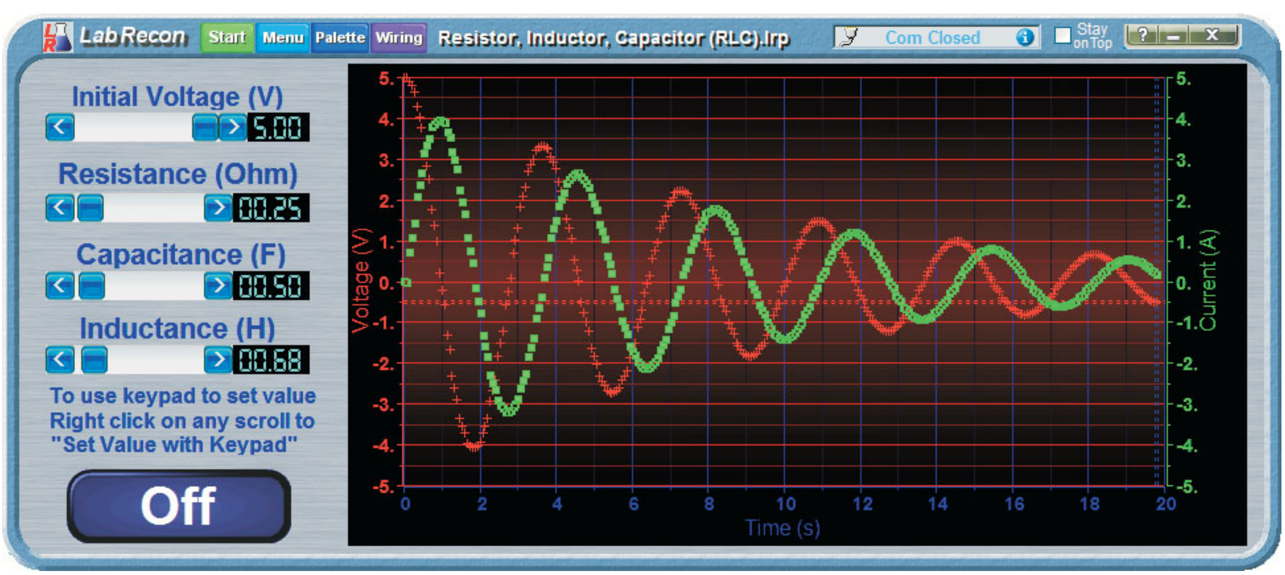
LRC Inductor, Capacitor, Resistor Circuit (continued)

$$\frac{VL}{L} = \frac{d^2Q}{dt^2} \qquad Qo = C \times Vo \qquad \frac{dQ}{dt} = \int \frac{d^2Q}{dt^2} \qquad Q = -1\int \frac{dQ}{dt} + Qo \qquad Vc = \frac{-Q}{C}$$



Ramp function is used to create time value, t, to plot values on XY Chart. The constant shown set as 1 determines the ramp rate. The lower constant shown as 20 sets the time period. The Task function clears the chart at the end of the time period. The Reset signal also resets the Integration function. A button was added to allow the simulation to be reset manually.

The waveform amplitude exhibits an exponential decay for the parameters shown. If the resistance is increased the oscillations will decay faster or no oscillation will occur.



LRC Inductor, Capacitor, Resistor Circuit (with Coulomb Friction)

In the previous LRC circuit the resistor (dashpot equivalent) was responsible for dissipating energy to dampen the action.

The force of the dashpot is a function of the velocity of it's piston deplacement, thus at low speeds the dashpot will offer little resistance. This characteristic is referred to as **fluid friction**.

For mechanical systems **coulomb friction** is often the dominant type of friction, whereas the force produced is constant and not dependent on speed.

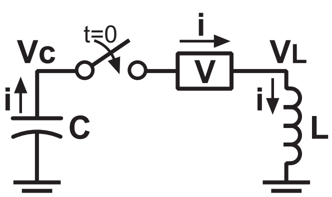
The reason why the dashpot, a **fluid friction** element, so often appears in textbook models is due to the analogy of its behavior to a resistor. This behavior is responsible for the exponential decay inherent in models employing a resistor.

In this example a slight modification is made to the Wiring Diagram to produce a model using coulomb friction. This will produce a linear decay of oscillations as opposed to an exponential decay.

The circuit/mechanical analogies used in this document used a voltage equivalence to force. Thus, to replace the fluid friction element with a Coulomb friction element the resistor will be replaced with a voltage source. However, as shown in the below equations, this voltage will be a DC voltage that will reverse polarity with the current. The voltage will also only be present when current is flowing.

An electrical approximation of this "friction" element could be constructed using diodes.

One implementation shown below uses two diodes in parallel with reversed biases. diodes are shown because they have lower forward voltages compared to typical silicon diodes. Multiple diodes can be used for higher voltages.

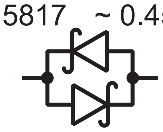


$$i = -C\frac{dV_c}{dt}$$

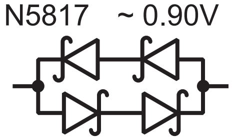
$$\left.\begin{array}{ll} V_r = F & \text{if } i > 0 \\ V_r = -F & \text{if } i < 0 \end{array}\right] \text{replacement for } V_r = R \times i$$

$$V_L = L\frac{di}{dt}$$

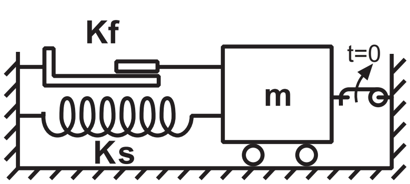
1N5817 ~ 0.45V



1N5817 ~ 0.90V



In the below system the dashpot is replaced with a Coulomb friction element. This results in a damping force that is constant as opposed to that from the dashpot, whereas the force is proportional to velocity.



spring stretched

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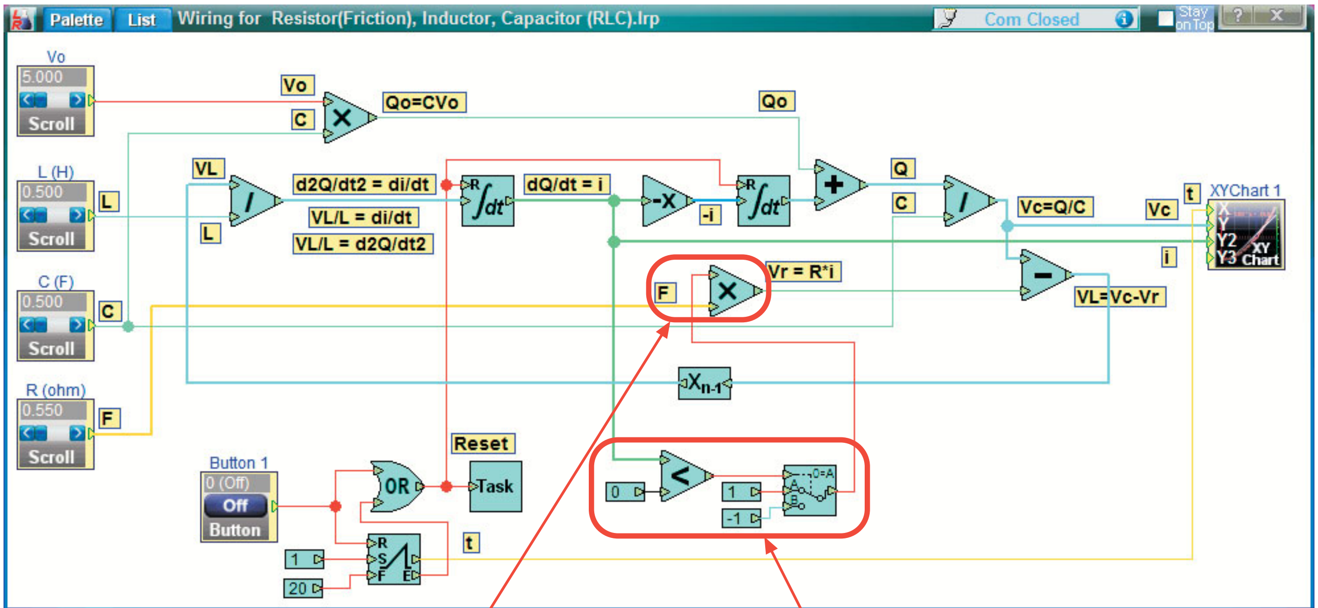
LRC Inductor, Capacitor, Resistor Circuit (with Coulomb Friction - continued)

The below screen shows the Wiring Diagram modified to produce a model using Coulomb friction.

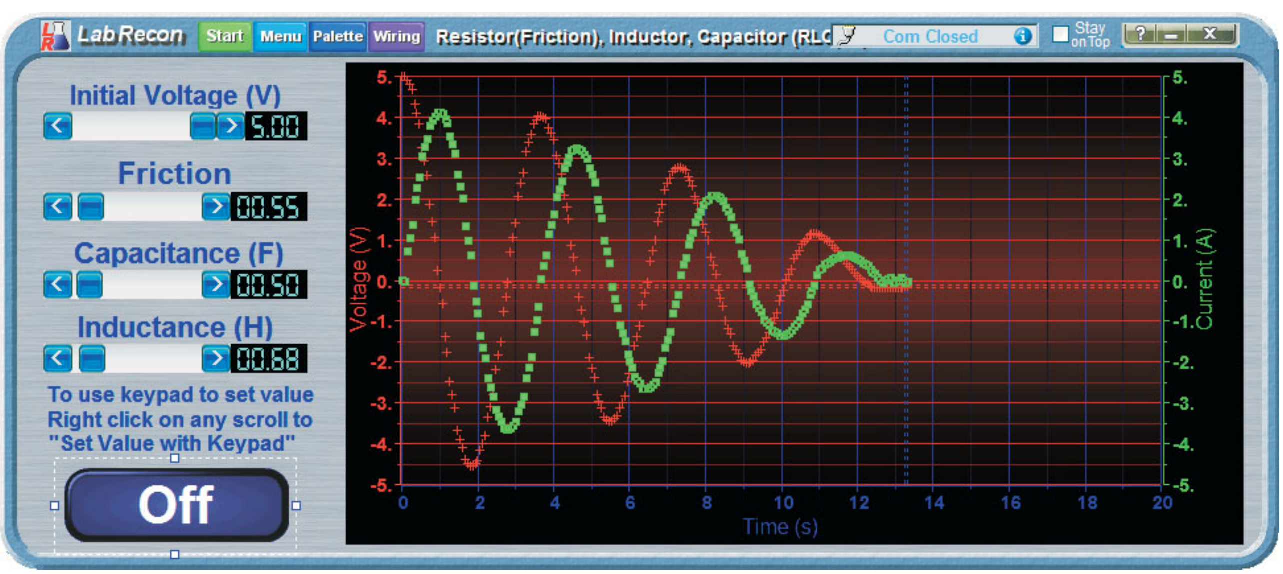


Originally the Multiply operated on both resistance and current to produce a voltage (force equivalent).

Now the Multiply is used only to insure the polarity of the friction value follows the polarity of the current.

This block produces the value of +1 or -1 depending on the polarity of the current.

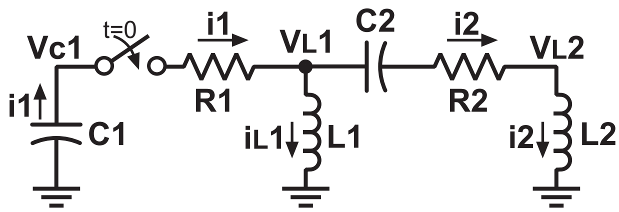
The waveform amplitude exhibits a linear decay with columb friction.



Dual LRC Inductor, Capacitor, Resistor Circuit (2nd order, 2 degrees of freedom)

The below circuit depicts two connected LRC circuits. The circuit and its mechanical analogy are 2DOF (2 degrees of freedom) systems. The circuit has two currents and the mechanical analogy has two velocities.

These systems have two time constants and thus can have two frequencies of natural oscillation.



equations for inductor, **L1**, current and voltage used to define the interdependancy between the two LRC loops

$$iL1 = i1 - i2$$

$$VL1 = VL2 + VR2 + VC2$$

equations in terms of current, **i**

$$i1 = -C1\,\frac{dVc1}{dt}$$

$$Vr1 = R1 \times i1$$

$$VL1 = L1\,\frac{diL1}{dt}$$

$$i2 = C2\,\frac{dVc2}{dt}$$

$$Vr2 = R2 \times i2$$

$$VL2 = L2\,\frac{diL2}{dt}$$

equations in terms of charge, **Q**

$$Vc1 = \frac{-Q1}{C1}$$

$$Vr1 = R1\,\frac{dQ1}{dt}$$

$$VL = L\,\frac{d^2Q}{dt^2}$$

$$Vc2 = \frac{Q2}{C2}$$

$$Vr2 = R2\,\frac{dQ2}{dt}$$

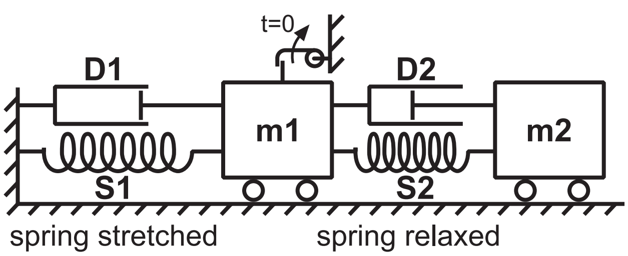
$$VL2 = L2\,\frac{d^2Q2}{dt^2}$$

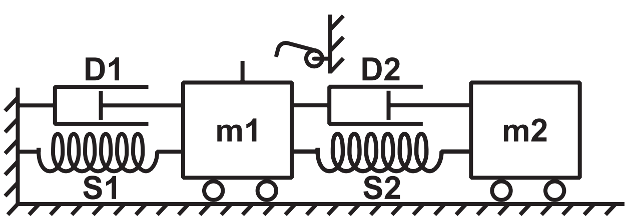
A possible mechanical analogy would be comprised of a spring, S1, (initially stretched) connected to a dashpot, D1, and a mass, m1. Another spring-dashpot-mass is connected with its spring, S2, initially relaxed.

When the hook is released the force of the spring accelerates the mass while also imparting a force on the 2nd spring-dashpot-mass.

The 2nd picture demonstrates a possible state soon after the hook is released, whereas m1 has been displaced to the left and m2 has been displaced to the left by a lesser amount and S2 has been stretched slightly.

There are many scenarios, depending on the parameters, in which both masses, or only one mass, or neither experience oscillatory behavior.





Mechanical		Electrical
force	⟷	voltage
position	⟷	charge
velocity	⟷	current
spring	⟷	capacitor
dashpot	⟷	resistor
mass	⟷	inductor
spring constant	⟷	1/capacitance
initial spring force	⟷	initial capacitor voltage

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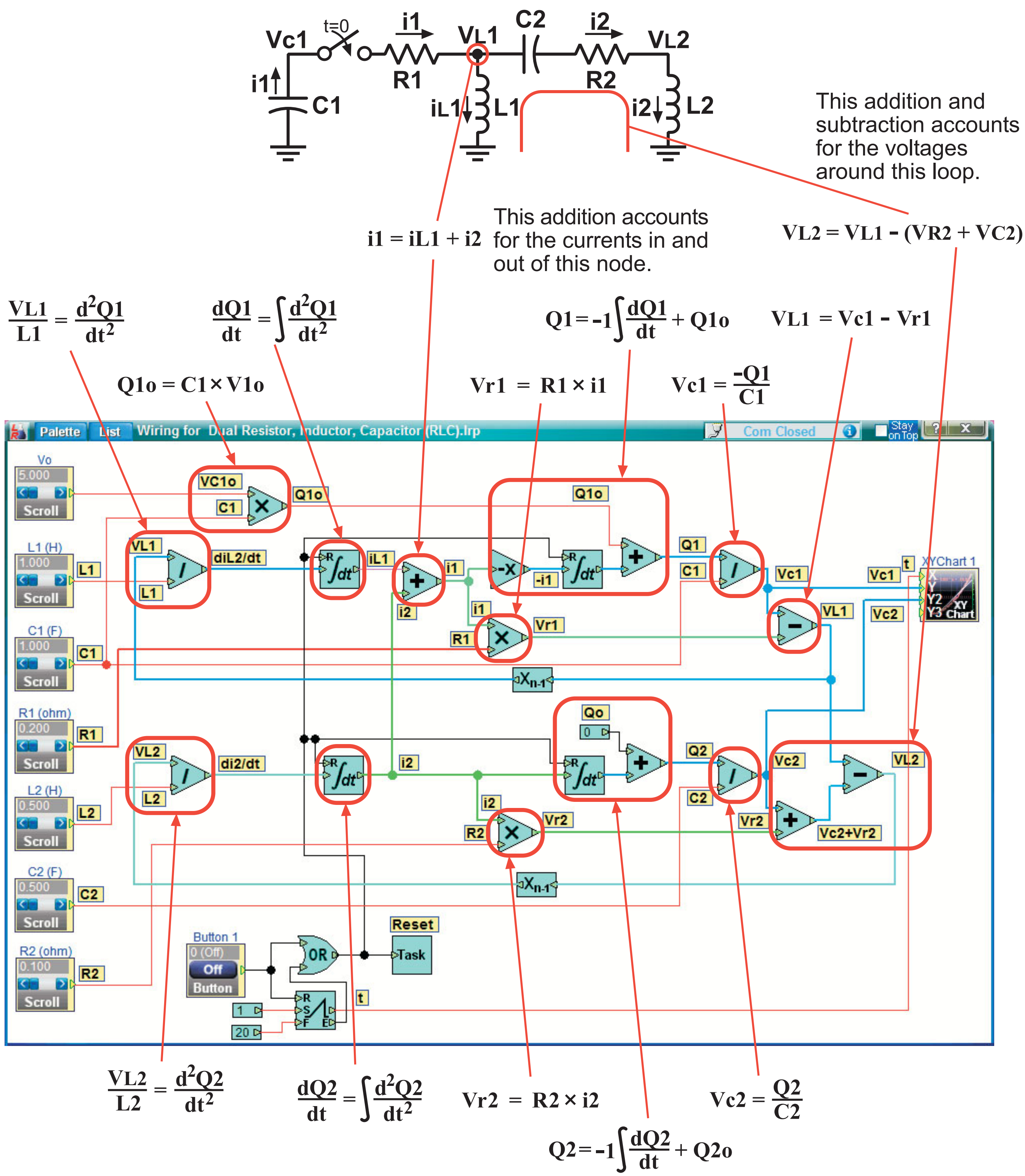
LRC Inductor, Capacitor, Resistor Circuit (continued)

Below is the circuit from the previous page with highlights to show how the dependancy between the two loops is implemented.



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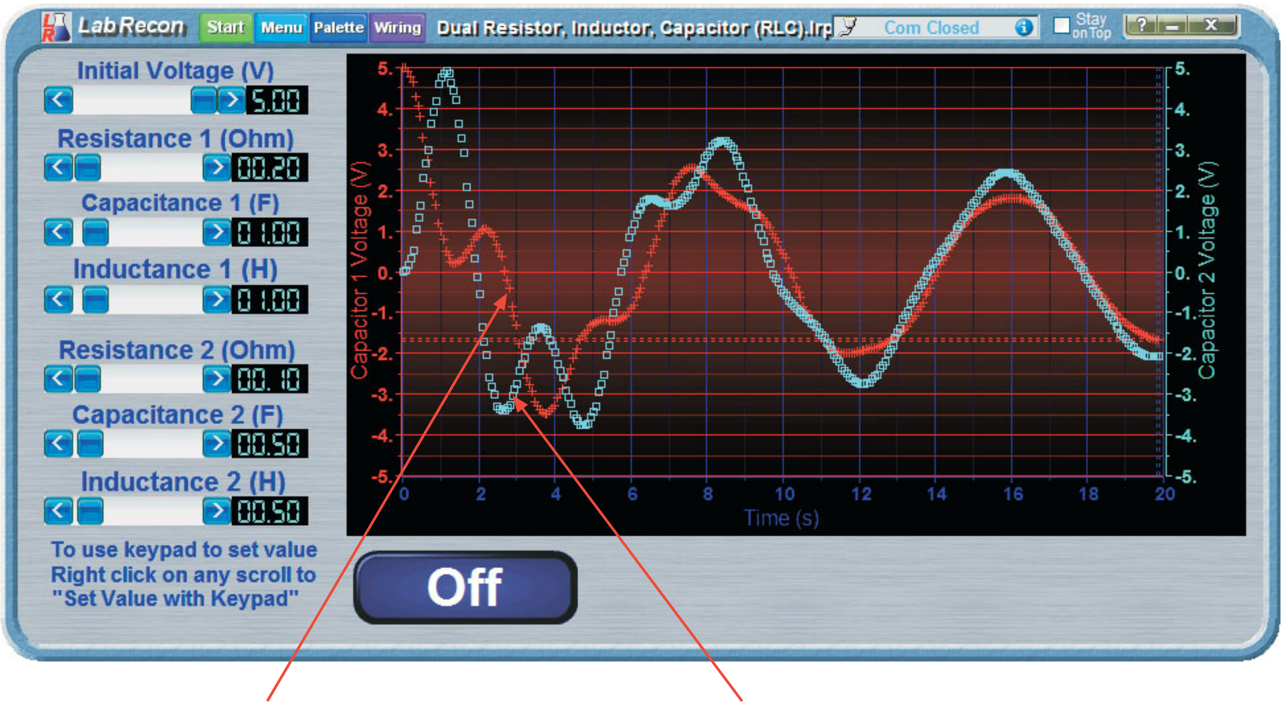
Dual LRC Inductor, Capacitor, Resistor Circuit (continued)

Below is the Panel showing the resultant capacitor voltages with an initial voltage of 5V on capacitor 1 and 0V on capacitor 2.



The red curve represents the 1st RLC loop, which has a natural frequency of ~0.25 Hz.

The blue curve represents the 2nd RLC loop, which has a natural frequency of ~0.50 Hz.

The scroll bars on the left can be used to adjust the circuit parameters. Adjusting capacitor and inductor values will effect the time constants and thus the frequency of any oscillation.

The resistor values can be adjusted to affect the damping. The screen shot above shows a result with both LRC loops underdamped to produce the oscillations. Increasing the resistances will allow overdamping to be demonstrated.